Scheme of the replica symmetry breaking for short- ranged Ising spin glass within the Bethe- Peierls method.

K. Walasek

Institute of Physics, Pedagogical University, Plac Słowiański 6,

65- 069 Zielona Góra, Poland.

Abstract

Within the Bethe- Peierls method the for short- ranged Ising spin glass ,recently formulated by Serva and Paladin, the equation for the spin glass parameter function near the transition to the paramagnetic phase has been carried out. The form of this equation is qualitatively similar to that for Sherrington- Kirpatrick model, but quantitatively the order parametr function depends of the dimension d of the system. In the case $d \to \infty$ one obtains well known Parisi solution.

75.10.Nr, 75.50.Lk

The study of spin glasses (SG)'s in finite dimensions is very active since it is still unclear if they share some the qualitative features of the mean- field theory of the model Sherrington-Kirkpatrick (SK) [1,2] However, there are recent investigations [3–6] which indicate difficulties to extend the mean field approximation (MFA) scenario to realistic spin glasses with short- range interaction and decide "a priori" which properties survive and which must be appropriately modified.

The one of the first attempt to go beyond MFA was an expansion of the SG order parameter for d -dimensional hypercubic lattice in $1/d$ [7]. It has turned out that in this case, qualitatively, the Parisi's ansatz [2] holds. Recently, in an interesting paper [8], an approach beyond the MFA has been achieved for an d- dimensional Ising SG model with short- range interactions on a real lattice using an extension of the Bethe- Peierls approximation (BPA) to the spin glass problem via the replica trick. This approach seems to be very promising to estabilish a direct contact with the results obtained by different authors for the infinite-ranged version and to controll possible deviations for short- ranged glasses from the well acquired MFA scenario. Quite recently [9] the Parisi's scheme has been investigated for the Ising SG with $S = 1/2$ using the generalized of the Bethe- Peierls method named by the authors "a variational approach" where finite clusters of spins interacts and the sample averaging is properly taken into account. The result for the free energy is qualitatively similar to that obtained in the frame of the MFA with some quantitatively modifications due to short- range order interactions. In particular in ref. [9] the two dimensional system is studied numerically on the one step replica symmetry breaking (RSB).

The aim of this Letter is to show that using only the Bethe- Peierls ansatz it is possible to obtain in an explicit form near the SG transition the form of the SG order parameter for all stages RSB. As usual the Hamiltonian of our system reads:

$$H = -\frac{1}{2}\Sigma_{i,j} J_{i,j}\sigma_i\sigma_j \quad , \tag{1}$$

where $\sigma_i = \pm 1$ and $J_{i,j}$ are random variables obeying dichotomic distribution, that is $J_{i,j} = \pm J$ with the equal probability for $+$ and $-$ sign. Using the replica trick combined with the

PBA (see ref. [8]) one obtains for the cluster consisting of the central spin and $2d$ its nearest neighbours the following effective Hamiltonian:

$$H_{\text{eff}} = \frac{-1}{\beta} \ln \left[\exp \left(\beta \sum_{i,\alpha} J_{0,i} \sigma_{0,\alpha} \sigma_{i,\alpha} \right) \right]_{\text{av}} - \frac{\beta J^2}{2} \sum_{\alpha \neq \alpha'} \sum_{i=1}^{2d} \mu_{\alpha,\alpha'} \sigma_{i,\alpha} \sigma_{i,\alpha'} \quad , \tag{2}$$

where $[\cdots]_{\text{av}}$ denotes sample averaging indices 0 and $i = 1 \cdots 2d$ refer to the central and lateral spins of the cluster, respectively, $\alpha \neq \alpha' = 1 \cdots n$ are replica indes with $n \to 0$ at the end of calculations. In eq. (2) $\mu_{\alpha,\alpha'}$'s describe the interaction betwen the "external world" and the lateral spins of the replicated cluster. Couplings $\mu_{\alpha,\alpha'}$ are calculated from the following equations:

$$\langle \sigma_{i,\alpha} \sigma_{i,\alpha'} \rangle = \langle \sigma_{0,\alpha} \sigma_{0,\alpha'} \rangle \quad , \tag{3}$$

where

$$\langle \cdots \rangle = \frac{\operatorname{Tr} \exp\left(-\beta H_{\text{eff}}\right) \cdots}{\operatorname{Tr} \exp\left(-\beta H_{\text{eff}}\right)} \quad . \tag{4}$$

It is easy to show the near below the SG transition $\mu_{\alpha,\alpha'}$ is a very small parameter and reach zero at the SG transition point and in paramagnetic phase. Therefore one can expect that the structure of $\mu_{\alpha,\alpha'}$'s is the same as of the SG order parameters $q_{\alpha,\alpha'} = \langle \sigma_\alpha \sigma_{\alpha'} \rangle$. Keeping this in mind, in order to recognize the form of the SG order parameter one expands the left and right hand side of (3) into $\mu_{\alpha,\alpha'}$ to the third order which is relevant to the study of the structure of $\mu_{\alpha,\alpha'}$ near the critical point. After a tedious but strighforward algebra we get the following equation:

$$\left(2\tau \mu_{\alpha,\alpha'} + A_d \sum_{\alpha_1} \mu_{\alpha,\alpha_1} \mu_{\alpha_1,\alpha'} + \frac{2}{3} B_d \mu^3_{\alpha,\alpha'} \right) = 0 \quad , \tag{5}$$

where $\tau = \left(T_c - T\right)/T_c$, $\beta_c = 1/T_c$ with T_c being the SG critical temperature (for the value of T_c see ref [8]), $A_d = \frac{\beta_c J}{(2d-1)^{1/2}}$ and $B_d = \frac{2\beta_c^3 J^3 d}{(2d-1)^{3/2}}$.

Now one introduces the Parisi scheme changing $\mu_{\alpha,\alpha'}$ to $\mu(x)$ with $0 < x < 1$ [2]. After this the eq. (5) takes the form:

$$2\tau \mu(x) - 2A_d \mu(x) \int_0^1 \mu(y)\, dy - A_d \int_0^x \left[\mu(x) - \mu(y)\right]^2 + \frac{2}{3} B_d \mu(x)^3 = 0 \tag{6}$$

After the standard procedure [2] one finds that

$$\mu\left(x\right)=\frac{A_d}{2B_d}x \quad , \tag{7}$$

for $0<x<x_1$. For $x_1\leq x\leq 1$ the parametr $\mu\left(x\right)$ reach a plateau and its value is

$$\mu\left(x\right)=\frac{A_d}{2B_d}x_1 \tag{8}$$

with

$$x_1=\frac{4\beta_c Jd}{\left(2d-1\right)^{1/2}}\tau \quad . \tag{9}$$

It is easy to show that for the infinite dimension ($d\rightarrow\infty$), after the aprioprate rescaling [8,10] $J\rightarrow J\sqrt{2d}$ and changing $\mu_{\alpha,\alpha'}$ to $2q_{\alpha,\alpha'}$ one obtains the well known Parisi scheme for the Sherrington- Kirpatrick model.

Therefore one concludes that in the frame of the BPA the Parisi ansatz holds for a real d-dimensional hypercubic lattices. If one assumes that the BPA aprroximation for the Ising SG (1) on the Bethe lattice is exact [11] one can infer that the Parisi's scheme is valid for that case.

The author acknowledge the financial support of the Polish Committee for Scientific Research (K. B. N.), Grant No 2 P03B 034 11 .

REFERENCES

[1] D. Sherrington and S. Kirkpatrick, Phys. Rev. Lett. **35**,1792 (1975).

[2] M. Mézard, G. Parisi, and M. Virasoro, *Spin Glass Theory and Beyond* (World Scientific, Singapore , 1988) and references therein.

[3] C. M. Newman and D. L. Stein, Phys. Rev. Lett. **76**, (1996) 515.

[4] A.K. Hartmann, preprint, cond- mat/9704187 (1997).

[5] G. Parisi, preprint, cond- mat/ 9603101 (1996).

[6] G. Parisi, preprint, cond- mat/ 9701034 (1997).

[7] A. Georges, M. Mézard and J. S. Yedidia, Phys. Rev. Lett. **64**, (1990) 2937.

[8] M. Serva and G. Paladin, Phys. Rev. E **54**, (1996) 4637.

[9] R. Baviera, M. Pasquini and M. Serva, J. Phys. A; Math. Gen. **31**, (1998) 4127 .

[10] K. Walasek *et al.*, J. Phys. A: Math.Gen. **31**, (1998) 5733.

[11] W. F. Wreszinski and S. R. A. Salinas, *Disorder and Competiton in Soluble Lattice Models, Series on Advances in Statistical Mechanics vol. 9* (World Scientific, 1993).